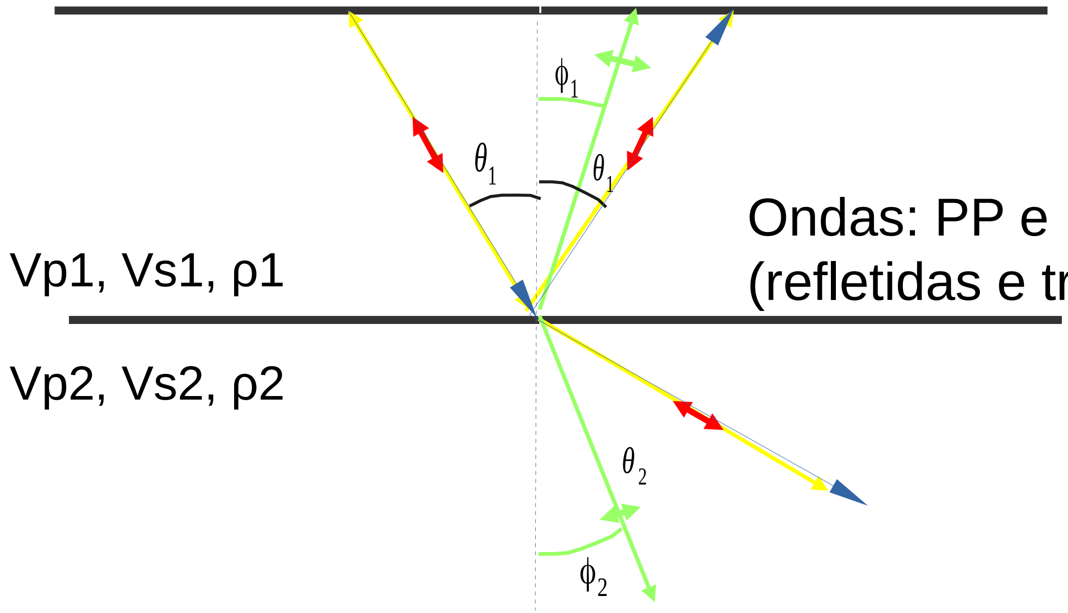
# AGG0232-Sísmica I
# 2023

## Aula 4
## Coeficientes de
## Reflexão e Transmissão
## (R e T)

Profa. Liliana A. Diogo

# **Ondas geradas numa interface que separa dois meios elásticos** (meios com rigidez → propagam-se ondas P e S)




Ondas: PP e PS (P convertida em S)
(refletidas e transmitidas)

Lei de Snell

$$p = \frac{\text{sen}(\theta_1)}{V_{p1}} = \frac{\text{sen}(\theta_2)}{V_{p2}} = \frac{\text{sen}(\phi_1)}{V_{s1}} = \frac{\text{sen}(\phi_2)}{V_{s2}}$$

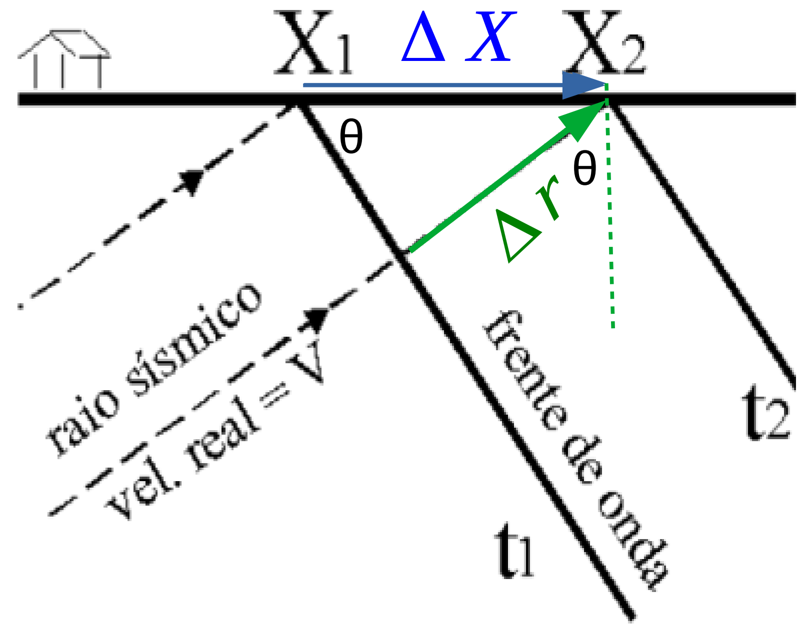
p= parâmetro do raio é constante durante toda a propagação

A Lei de Snell trata dos aspectos geométricos. A partição da energia na interface é equacionada pelos Coeficientes de **R**eflexão e **T**ransmissão (eqs. De Zoeppritz)

# Velocidade medida x Velocidade Real

$$V_{medida} = \frac{X_2 - X_1}{t_2 - t_1} = \frac{\Delta X}{\Delta t}$$

$$V_{medida} = V_{aparente} = V_{ap}$$




$$V_{real} = \frac{\Delta r}{\Delta t} \qquad sen(\theta) = \frac{\Delta r}{\Delta X}$$

$$V_{real} = \frac{\Delta X\, sen(\theta)}{\Delta t} = sen(\theta)\, V_{medida}$$

$$V_{ap} = \frac{V_r}{sen(\theta)}$$

$$\frac{1}{V_{ap}} = \frac{sen(\theta)}{V_r}$$

Vagarosidade (Slowness)

# Conceitualmente, a teoria para a dedução dos Coeficientes R-T

1) Teoria da Elasticidade:
- Parâmetros elásticos (definições)
- Lei de Hooke: esforço = **C** deformação
- Tensor dos esforços
- Tensor das deformações
- **C** = Matriz (6X6) das constantes elásticas

2) Dedução da Equação da onda:
Análise dos esforços (força/área) resultantes por unidade de volume
2a. Lei de newton (F=m.a)  (m=ρv)

3) Expressões para a solução da eq. de onda: aproximação para onda planas

4) Condições de contorno (continuidade) na interface dos esforços e deformações, normais e de cisalhamento => 4 equações, a partir das quais podem ser obtidas expressões para os coeficientes de Reflexão e Transmissão Rpp, Tpp, Rps e Tps (Equações de Zoeppritz (1919))

$$R_{PP}=\frac{A_{refletida}}{A_{incidente}}$$

$$R_{PP}=\frac{A_1}{A_0}$$

$$T_{PP}=\frac{A_{transmitida}}{A_{incidente}}$$

$$T_{PP}=\frac{A_2}{A_0}$$

Falta inserir um desenho aqui

# Equações de Zoeppritz reescritas por Aki e Richards (1980)

$$\underline{\mathbf{Q}} = \begin{pmatrix} \grave{P}\acute{P} & \grave{S}\acute{P} & \acute{P}\acute{P} & \acute{S}\acute{P} \\ \grave{P}\acute{S} & \grave{S}\acute{S} & \acute{P}\acute{S} & \acute{S}\acute{S} \\ \grave{P}\grave{P} & \grave{S}\grave{P} & \acute{P}\grave{P} & \acute{S}\grave{P} \\ \grave{P}\grave{S} & \grave{S}\grave{S} & \acute{P}\grave{S} & \acute{S}\grave{S} \end{pmatrix} = \underline{\mathbf{P}}^{-1}\underline{\mathbf{R}}$$

where $\underline{\mathbf{P}}$ is the matrix

$$\begin{pmatrix} -\sin\Theta_1 & -\cos\Phi_1 & \sin\Theta_2 & \cos\Phi_2 \\ \cos\Theta_1 & -\sin\Phi_1 & \cos\Theta_2 & -\sin\Phi_2 \\ 2\rho_1 V_{S1}\sin\Phi_1\cos\Theta_1 & \rho_1 V_{S1}(1-2\sin^2\Phi_1) & 2\rho_2 V_{S2}\sin\Phi_2\cos\Theta_2 & \rho_2 V_{S2}(1-2\sin^2\Phi_2) \\ -\rho_1 V_{P1}(1-2\sin^2\Phi_1) & \rho_1 V_{S1}\sin 2\Phi_1 & \rho_2 V_{P2}(1-2\sin^2\Phi_2) & -\rho_2 V_{S2}\sin 2\Phi_2 \end{pmatrix}$$

and $\underline{\mathbf{R}}$ is the matrix

$$\begin{pmatrix} \sin\Theta_1 & \cos\Phi_1 & -\sin\Theta_2 & -\cos\Phi_2 \\ \cos\Theta_1 & -\sin\Phi_1 & \cos\Theta_2 & -\sin\Phi_2 \\ 2\rho_1 V_{S1}\sin\Phi_1\cos\Theta_1 & \rho_1 V_{S1}(1-2\sin^2\Phi_1) & 2\rho_2 V_{S2}\sin\Phi_2\cos\Theta_2 & \rho_2 V_{S2}(1-2\sin^2\Phi_2) \\ \rho_1 V_{P1}(1-2\sin^2\Phi_1) & -\rho_1 V_{S1}\sin 2\Phi_1 & -\rho_2 V_{P2}(1-2\sin^2\Phi_2) & \rho_2 V_{S2}\sin 2\Phi_2 \end{pmatrix}.$$

# Equações de Zoeppritz na forma matricial

$$\begin{bmatrix} \sin\theta_i & \cos\phi_r & -\sin\theta_t & \cos\phi_t \\ \cos\theta_i & -\sin\phi_r & \cos\theta_t & \sin\phi_t \\ \sin 2\theta_i & a_1\cos 2\phi_r & b_1 \sin 2\theta_t & -c_1\cos 2\phi_t \\ \cos 2\phi_r & -a_2 \sin 2\phi_r & -b_2\cos 2\phi_t & -c_2\sin 2\phi_t \end{bmatrix}$$

$$\times \begin{bmatrix} \boxed{\begin{matrix} R_{PP} \\ R_{PS} \\ T_{PP} \\ T_{PS} \end{matrix}} \end{bmatrix} = \begin{bmatrix} -\sin\theta_i \\ \cos\theta_i \\ \sin 2\theta_i \\ -\cos 2\phi_r \end{bmatrix}. \quad (3.163)$$

The system of equations in (3.163) can be solved numerically on a computer, for each set of parameters.

We have also given the solution for reflection and transmission in terms of slownesses, which result in more-compact expressions. Readers are referred to Box 3.4 for the algebraic solution.

*Ikelle e Amundsen,1995*
*Introduction to Petroleum Seismology*

## BOX 3.4: R/T COEFFICIENTS IN TERMS OF SLOWNESS: SOLID-SOLID INTERFACE

This box gives the reflection and transmission coefficients at a solid-solid interface in terms of horizontal slowness $p$ and vertical slownesses:

$$q_{P1} = \sqrt{V_{P1}^{-2} - p^2} \quad : \quad \text{P-wave top layer;}$$

$$q_{S1} = \sqrt{V_{S1}^{-2} - p^2} \quad : \quad \text{S-wave top layer;}$$

$$q_{P2} = \sqrt{V_{P2}^{-2} - p^2} \quad : \quad \text{P-wave bottom layer; and}$$

$$q_{S2} = \sqrt{V_{S2}^{-2} - p^2} \quad : \quad \text{S-wave bottom layer.}$$

Introduce

$$d_1 = 2p^2\Delta\mu(q_{P1} - q_{P2}) + (\rho_1 q_{P2} + \rho_2 q_{P1});$$

$$d_2 = 2p^2\Delta\mu(q_{S1} - q_{S2}) + (\rho_1 q_{S2} + \rho_2 q_{S1});$$

$$d_3 = p[2\Delta\mu(q_{P1}q_{S2} + p^2) - \Delta\rho];$$

$$d_4 = p[2\Delta\mu(q_{P2}q_{S1} + p^2) - \Delta\rho];$$

$$c_1 = 2p^2\Delta\mu(q_{P1} + q_{P2}) - (\rho_1 q_{P2} - \rho_2 q_{P1});$$

$$c_2 = -[2p^2\Delta\mu(q_{S1} + q_{S2}) - (\rho_1 q_{S2} - \rho_2 q_{S1})];$$

$$c_3 = -p[2\Delta\mu(q_{P1}q_{S2} - p^2) + \Delta\rho];$$

$$c_4 = -p[2\Delta\mu(q_{P2}q_{S1} - p^2) + \Delta\rho].$$

with contrast parameters $\Delta\mu = \mu_1 - \mu_2$ and $\Delta\rho = \rho_1 - \rho_2$. The reflection coefficients for downward-traveling incident plane waves in the lower solid are

$$R_{PP} = \frac{c_1 d_2 - c_3 d_4}{d_1 d_2 + d_4 d_3};$$

$$R_{PS} = -\left(\frac{V_{P1}}{V_{S1}}\right)\frac{c_3 d_1 + c_1 d_3}{d_1 d_2 + d_4 d_3}.$$

$$R_{SP} = \left(\frac{V_{S1}}{V_{P1}}\right)\frac{c_4 d_2 - c_2 d_4}{d_1 d_2 + d_4 d_3};$$

$$R_{SS} = -\frac{c_2 d_1 + c_4 d_3}{d_1 d_2 + d_4 d_3}.$$

The corresponding transmission coefficients are

$$T_{PP} = \left(\frac{V_{P1}}{V_{P2}}\right)\frac{2\rho_1 q_{P1} d_2}{d_1 d_2 + d_4 d_3};$$

$$T_{PS} = -\left(\frac{V_{P1}}{V_{S2}}\right)\frac{2\rho_1 q_{P1} d_4}{d_1 d_2 + d_4 d_3}.$$

$$T_{SP} = \left(\frac{V_{S1}}{V_{P2}}\right)\frac{2\rho_1 q_{S1} d_3}{d_1 d_2 + d_4 d_3};$$

$$T_{SS} = \left(\frac{V_{S1}}{V_{S2}}\right)\frac{2\rho_1 q_{S1} d_1}{d_1 d_2 + d_4 d_3}.$$

The reflection coefficients for upward traveling incident plane waves in the upper solid are

$$R_{PP}^{(U)} = -\frac{d_2 c_1 + d_3 c_4}{d_1 d_2 + d_4 d_3};$$

$$R_{PS}^{(U)} = -\left(\frac{V_{P2}}{V_{S2}}\right)\frac{d_4 c_1 - d_1 c_4}{d_1 d_2 + d_4 d_3}.$$

$$R_{SP}^{(U)} = -\left(\frac{V_{S2}}{V_{P2}}\right)\frac{d_2 c_3 + d_3 c_2}{d_1 d_2 + d_4 d_3};$$

$$R_{SS}^{(U)} = -\frac{d_4 c_3 - d_1 c_2}{d_1 d_2 + d_4 d_3}.$$

The corresponding transmission coefficients are

$$T_{PP}^{(U)} = \left(\frac{V_{P2}}{V_{P1}}\right)\frac{2\rho_2 q_{P2} d_2}{d_1 d_2 + d_4 d_3};$$

$$T_{PS}^{(U)} = \left(\frac{V_{P2}}{V_{S1}}\right)\frac{2\rho_2 q_{P2} d_3}{d_1 d_2 + d_4 d_3}.$$

$$T_{SP}^{(U)} = -\left(\frac{V_{S2}}{V_{P1}}\right)\frac{2\rho_2 q_{S2} d_4}{d_1 d_2 + d_4 d_3};$$

$$T_{SS}^{(U)} = \left(\frac{V_{S2}}{V_{S1}}\right)\frac{2\rho_2 q_{S2} d_1}{d_1 d_2 + d_4 d_3}.$$

# Coeficientes de Reflexão (R) e Transmissão (T)

## CASO ESPECIAL: Incidência Normal à interface

$$R_{PP} = \frac{\rho_2 V_{P2} - \rho_1 V_{P1}}{\rho_2 V_{P2} + \rho_1 V_{P1}} \qquad T_{PP} = \frac{2\rho_1 V_{P1}}{\rho_2 V_{P2} + \rho_1 V_{P1}} \qquad R_{PS} = 0 \;\; e \;\; T_{PS} = 0$$

*Intuitivamente, $R_{PS}$ e $T_{PS}$ são nulos para incidência normal, pois não há tensão de cisalhamento para uma onda P com incidência normal à interface.*

Impedância Acústica: $Z_i = \rho_i V_{Pi}$

$$\boxed{R_{PP} = \frac{Z_2 - Z_1}{Z_2 + Z_1}} \qquad \boxed{T_{PP} = \frac{2 Z_1}{Z_2 + Z_1}}$$

Essas expressões podem ser usadas para analisar o comportamento da onda P em ambientes de várias camadas, para pequenos ângulos de incidência (próximos da normal)

**Coeficientes de Reflexão (R) e Transmissão (T)**

**CASO ESPECIAL: Meios Acústicos** (só propagam onda P)

$$R_{PP} = \frac{\rho_2 V_{P2} \cos(\theta_1) - \rho_1 V_{P1} \cos(\theta_2)}{\rho_2 V_{P2} \cos(\theta_1) + \rho_1 V_{P1} \cos(\theta_2)}$$

$$T_{PP} = \frac{2\rho_1 V_{P1} \cos(\theta_1)}{\rho_2 V_{P2} \cos(\theta_1) + \rho_1 V_{P1} \cos(\theta_2)}$$

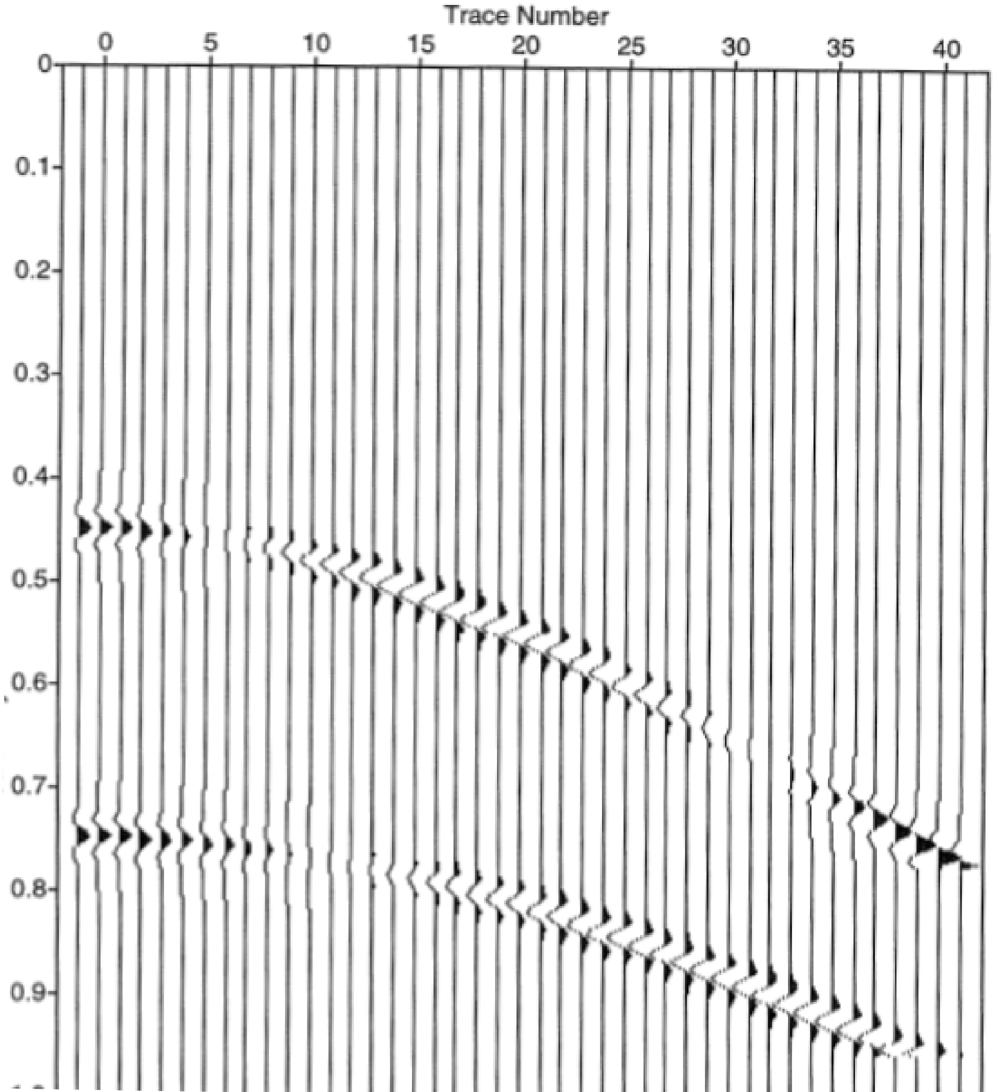
# Coeficientes de Reflexão (R) e Transmissão (T)

## CASO ESPECIAL: Meios Acústicos (só propagam onda P)

$$R_{PP}=\frac{\rho_2 V_{P2}\cos(\theta_1)-\rho_1 V_{P1}\cos(\theta_2)}{\rho_2 V_{P2}\cos(\theta_1)+\rho_1 V_{P1}\cos(\theta_2)}$$

$$T_{PP}=\frac{2\rho_1 V_{P1}\cos(\theta_1)}{\rho_2 V_{P2}\cos(\theta_1)+\rho_1 V_{P1}\cos(\theta_2)}$$

O que ocorre na expressão de Rpp quando para ângulos de incidência acima do ângulo crítico ???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5
10
15
20
25
30
35
40
0
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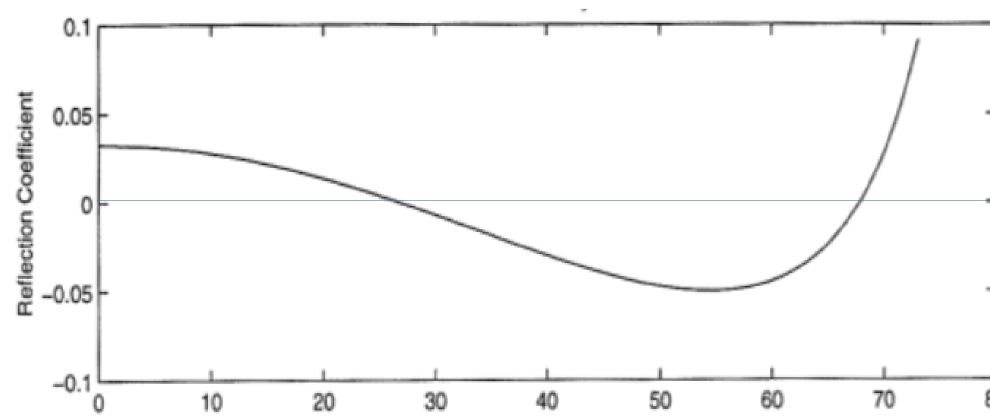
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9


Reflection Coefficient
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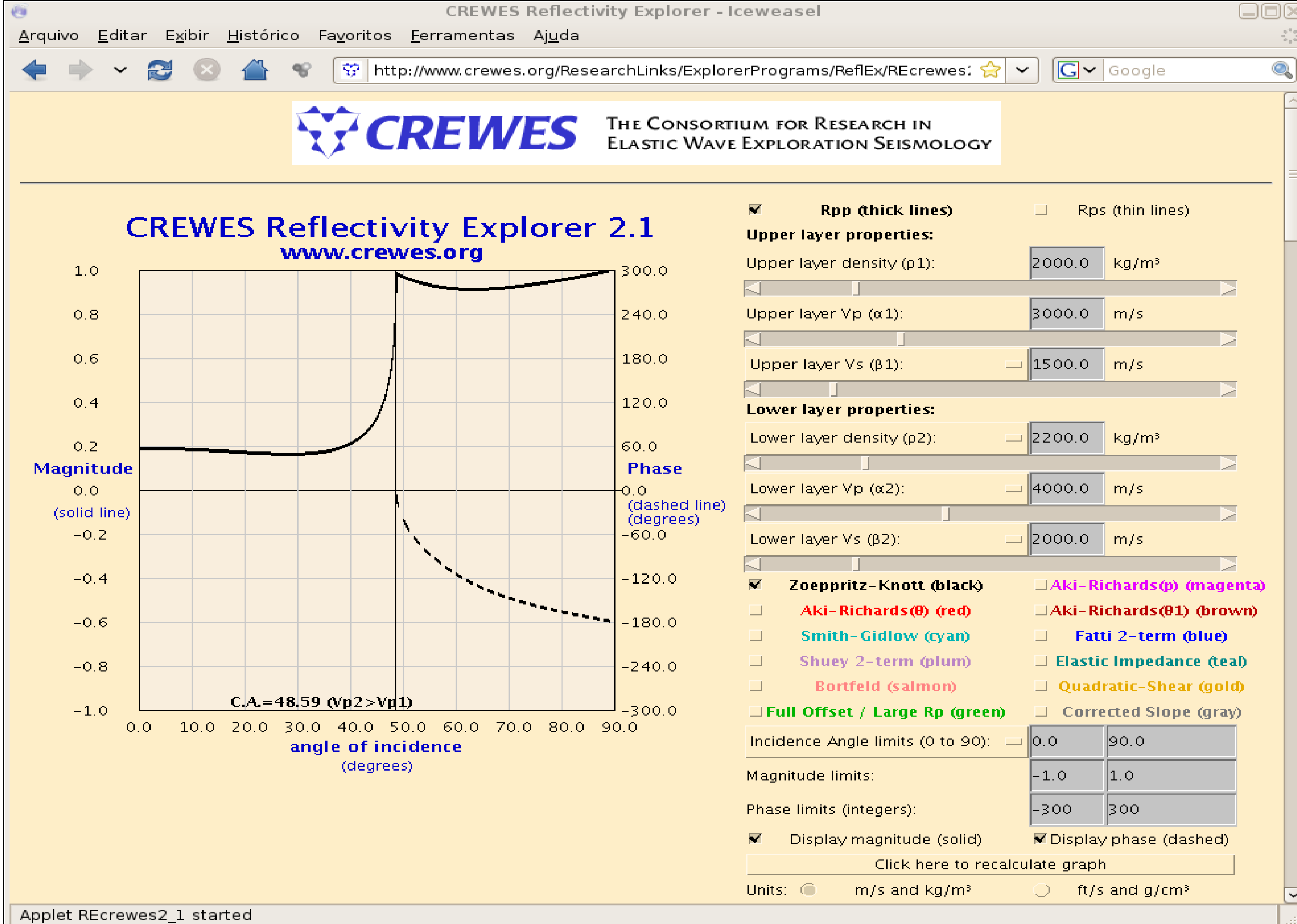
0.05
0
-0.05
-0.1
0
10
20
30
40
50
60
70
80


CREWES Reflectivity Explorer - Iceweasel
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http://www.crewes.org/ResearchLinks/ExplorerPrograms/ReflEx/REcrewes2
Google
CREWES
The Consortium for Research in Elastic Wave Exploration Seismology
CREWES Reflectivity Explorer 2.1
www.crewes.org
1.0
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Magnitude
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120.0
60.0
Phase
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(dashed line)
(degrees)
-60.0
-120.0
-180.0
-240.0
-300.0
C.A.=48.59 (Vp2>Vp1)
0.0 10.0 20.0 30.0 40.0 50.0 60.0 70.0 80.0 90.0
angle of incidence
(degrees)
Rpp (thick lines)
Rps (thin lines)
Upper layer properties:
Upper layer density (ρ1): 2000.0 kg/m³
Upper layer Vp (α1): 3000.0 m/s
Upper layer Vs (β1): 1500.0 m/s
Lower layer properties:
Lower layer density (ρ2): 2200.0 kg/m³
Lower layer Vp (α2): 4000.0 m/s
Lower layer Vs (β2): 2000.0 m/s
Zoeppritz-Knott (black)
Aki-Richards(p) (magenta)
Aki-Richards(θ) (red)
Aki-Richards(θ1) (brown)
Smith-Gidlow (cyan)
Fatti 2-term (blue)
Shuey 2-term (plum)
Elastic Impedance (teal)
Bortfeld (salmon)
Quadratic-Shear (gold)
Full Offset / Large Rp (green)
Corrected Slope (gray)
Incidence Angle limits (0 to 90): 0.0 90.0
Magnitude limits: -1.0 1.0
Phase limits (integers): -300 300
Display magnitude (solid)
Display phase (dashed)
Click here to recalculate graph
Units: m/s and kg/m³ ft/s and g/cm³
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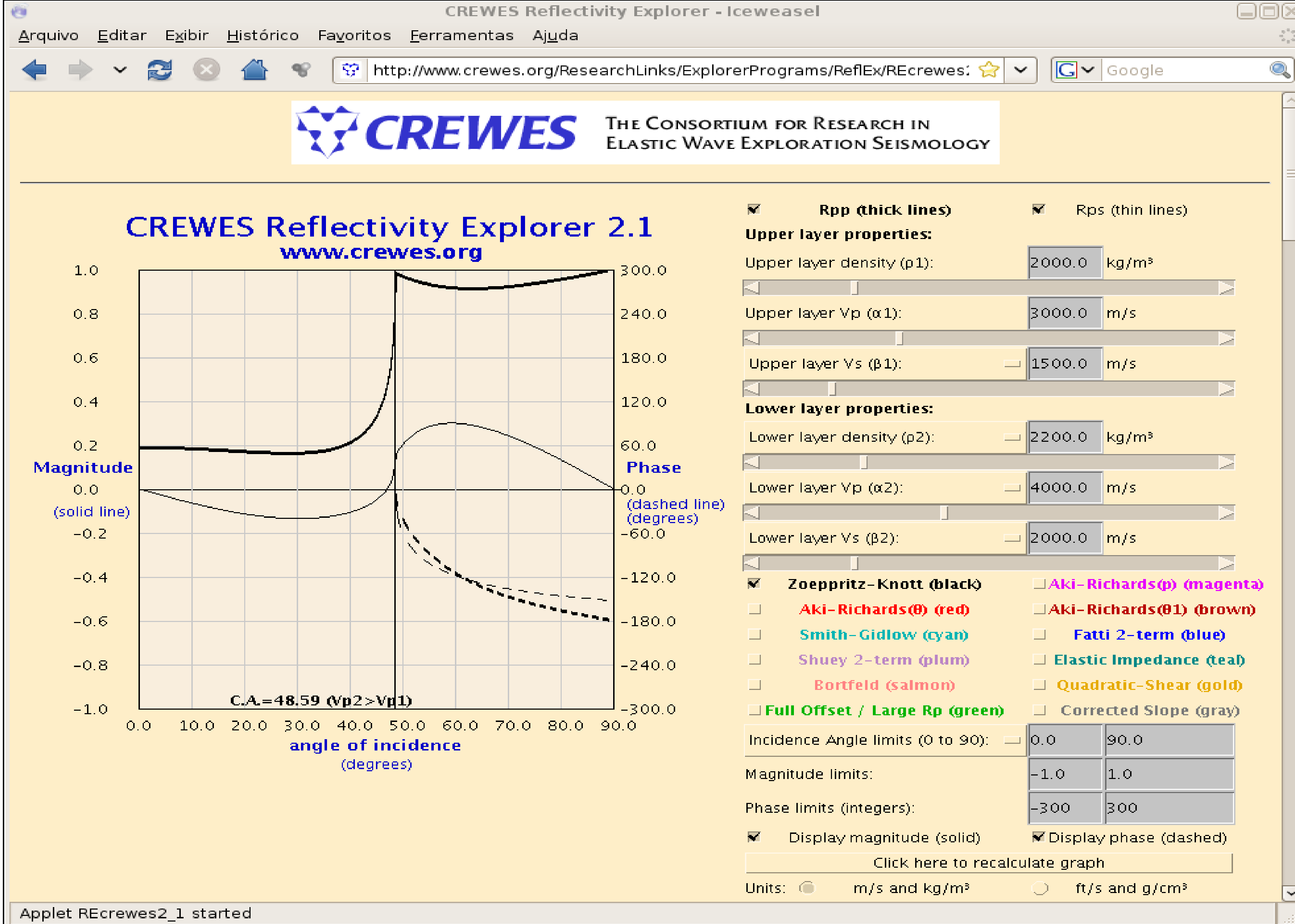
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CREWES Reflectivity Explorer 2.1
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Magnitude
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Display magnitude (solid)
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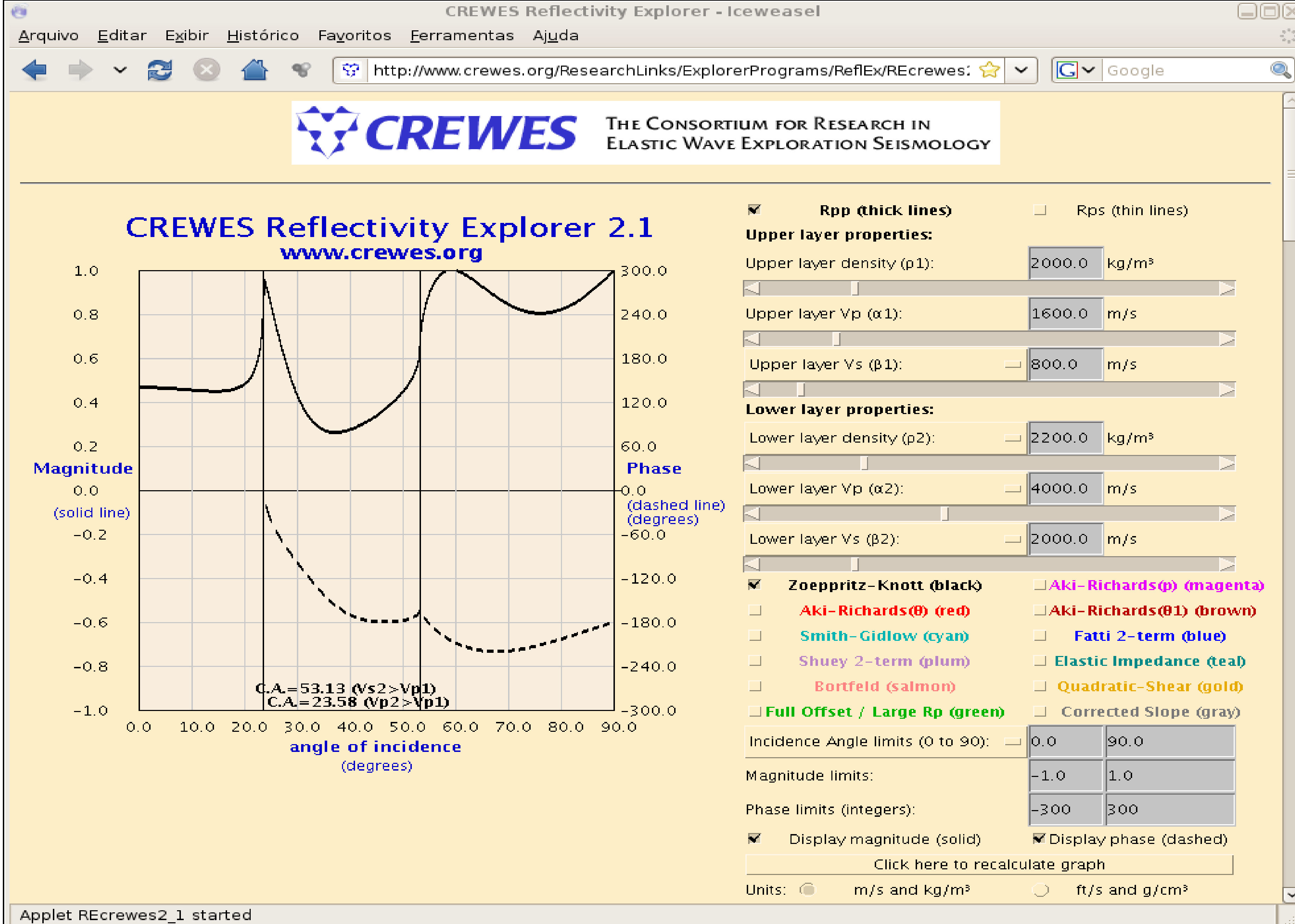
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Magnitude
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Phase
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180.0
120.0
60.0
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-60.0
-120.0
-180.0
-240.0
-300.0
C.A.=53.13 (Vs2>Vp1)
C.A.=23.58 (Vp2>Vp1)
0.0 10.0 20.0 30.0 40.0 50.0 60.0 70.0 80.0 90.0
angle of incidence
(degrees)
Rpp (thick lines)
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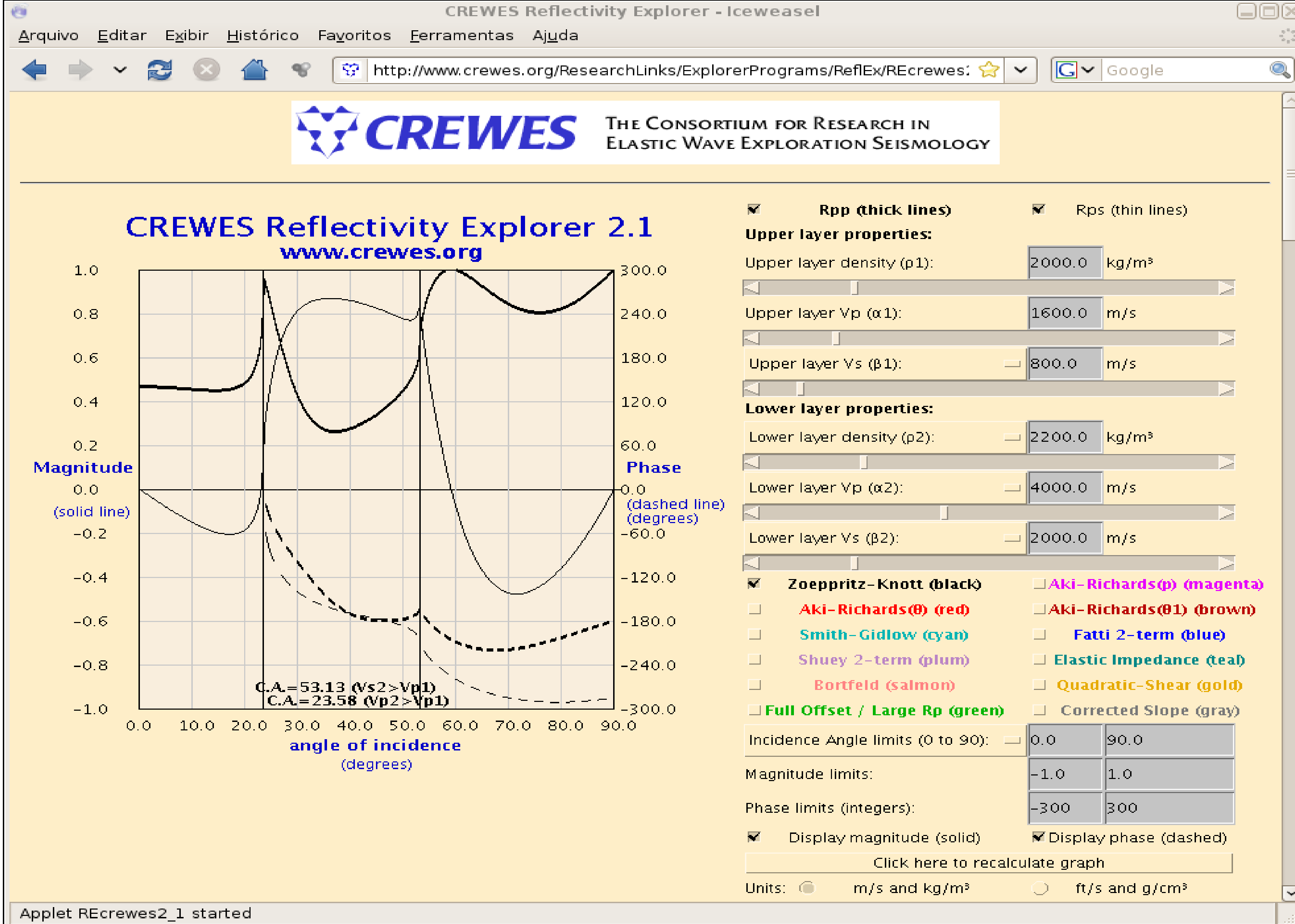
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Magnitude
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# EXERCÍCIOS

# Coeficientes de Reflexão (R) e Transmissão (T)

## CASO ESPECIAL: Incidência Normal à interface

$$R_{PP}=\frac{\rho_2 V_{P2}-\rho_1 V_{P1}}{\rho_2 V_{P2}+\rho_1 V_{P1}} \qquad T_{PP}=\frac{2\rho_1 V_{P1}}{\rho_2 V_{P2}+\rho_1 V_{P1}} \qquad R_{PS}=0 \quad e \quad T_{PS}=0$$

*Intuitivamente, $R_{PS}$ e $T_{PS}$ são nulos para incidência normal, pois não há tensão de cisalhamento para uma onda P com incidência normal à interface.*

Impedância Acústica: $Z_i=\rho_i V_{Pi}$

$$\boxed{R_{PP}=\frac{Z_2-Z_1}{Z_2+Z_1}} \qquad \boxed{T_{PP}=\frac{2Z_1}{Z_2+Z_1}}$$

$$\boxed{R_{PP}=\frac{A_{refletida}}{A_{incidente}}} \qquad \boxed{R_{PP}=\frac{A_1}{A_0}}$$

$$\boxed{T_{PP}=\frac{A_{transmitida}}{A_{incidente}}} \qquad \boxed{T_{PP}=\frac{A_2}{A_0}}$$

Essas expressões podem ser usadas para analisar o comportamento da onda P em ambientes de várias camadas, para pequenos ângulos de incidência (próximos da normal)