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Prova 1

Correção da prova

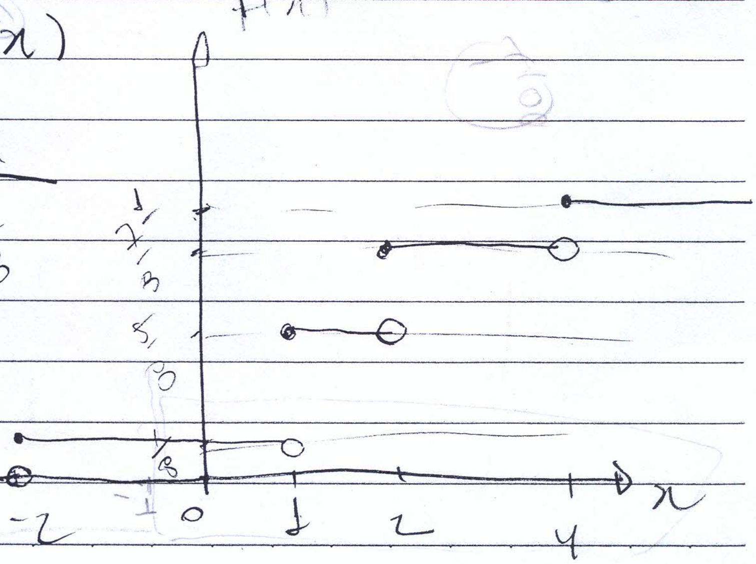
1) $E[X] = \sum_i x_i P[x_i]$

-2	1	2	4	
0	$\frac{1}{8}$	$\frac{5}{8}$	$\frac{7}{8}$	1

$$F(x) = \begin{cases} 0, & \text{se } x < -2 \\ \frac{1}{8}, & \text{se } -2 \le x < 1 \\ \frac{5}{8}, & \text{se } 1 \le x < 2 \\ \frac{7}{8}, & \text{se } 2 \le x < 4 \\ 1, & \text{se } x \ge 4 \end{cases}$$

$F(x) = P(X(w) \le x)$

x	-2	1	2	4
$P(x)$	$\frac{1}{8}$	$\frac{1}{2} = \frac{4}{8}$	$\frac{2}{8} = \frac{1}{4}$	$\frac{1}{8}$



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$$E[x] = -2\cdot\frac{1}{8} + 1\cdot\frac{1}{2} + 2\cdot\frac{1}{4} + 4\cdot\frac{1}{8} = \frac{5}{4}$$

$$Var[x] = E[(X-\mu)^2]$$

$$\mu = E[x]$$

$$Var[x] = E[x^2] - E[x]^2$$

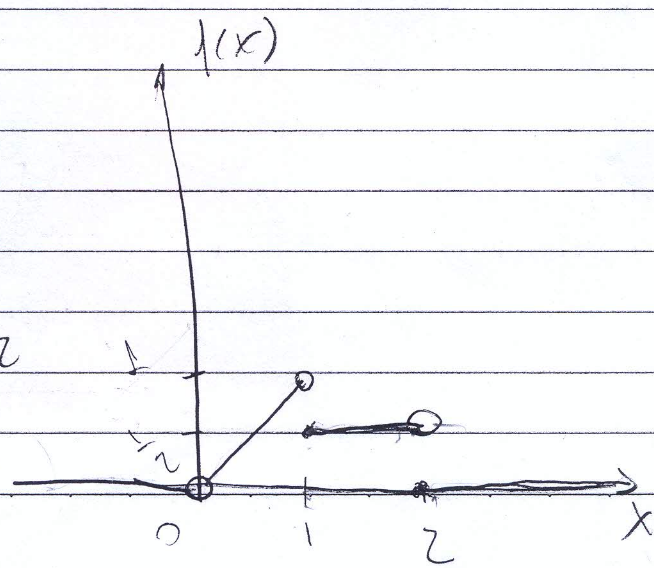
$$E[X^2] = \sum_i x_i^2 \, P(x_i)$$

$$E[x^2] = (-2)^2\frac{1}{8} + 1^2\cdot\frac{1}{2} + 2^2\cdot\frac{1}{4} + 4^2\cdot\frac{1}{8} = 4$$

$$var[x] = 4 - \left(\frac{5}{4}\right)^2$$

② $X \to$ VA contínua com densidade por:

$$f(x) = \begin{cases} 0, & x < 0 \\ x, & 0 \le x < 1 \\ 1/2, & 1 \le x < 2 \\ 0, & x \geq 2 \end{cases}$$



$$\int_{-\infty}^{+\infty} f(x)\,dx = 1 \qquad f(x) \geq 0 \quad , \forall x \in \mathbb{R}$$

$$F(x) = \int_{-\infty}^{x} f(y)\,dy$$

$F(x)$ = função de distribuição

$f(x)$ = função densidade

(i) $x < 0 \Rightarrow F(x) = \int_{-\infty}^{x} f(y)\,dy$

$$= \int_{-\infty}^{x} 0\,dy = 0 = 0 \int_{-\infty}^{x} dy = 0$$

$\therefore F(x) = 0$ p/ $x < 0$

(ii) $0 \leq x < 1$

$$F(x) = \int_{-\infty}^{x} f(y)\,dy = \int_{-\infty}^{0} f(y)\,dy + \int_{0}^{x} f(y)\,dy = 0 + \int_{0}^{x} y\,dy$$

$$= 0 + \left.\frac{y^2}{2}\right|_{0}^{x} = \frac{x^2}{2} \quad \therefore F(x) = \frac{x^2}{2} \text{ p/ } 0 \leq x < 1$$

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(iii) $1 \le x < 2$

$$F(x) = \int_{-\infty}^{x} f(y)\,dy = \int_{-\infty}^{0} f(y)\,dy + \int_{0}^{1} f(y)\,dy + \int_{1}^{x} f(y)\,dy =$$

$$+ \int_{0}^{1} y\,dy + \int_{1}^{x} \frac{1}{2}\,dy = 0 + \frac{y^2}{2}\Big|_{0}^{1} + \frac{1}{2}y\Big|_{1}^{x}$$

$$= 0 + \frac{1}{2} + \frac{1}{2}(x-1) = \frac{1}{2} + \frac{x}{2} - \frac{1}{2} = \frac{x}{2}$$

$\therefore F(x) = \frac{x}{2}$, για $1 \le x < 2$

iv) $x \geq 2$

$$F(X) = \int_{-\infty}^{x} f(y)\,dy = \int_{-\infty}^{0} f(y)\,dy + \int_{0}^{1} f(y)\,dy +$$

$$+ \int_{1}^{2} f(y)\,dy + \int_{2}^{x} f(y)\,dy =$$

$$= 0 + \int_{0}^{1} y\,dy + \int_{1}^{2} \frac{1}{2}\,dy + \int_{2}^{\infty} 0\,dy = 0 + \frac{1}{2} + \frac{1}{2}y\Big|_{1}^{2} + 0$$

$$= \frac{1}{2} + \frac{1}{2}(2-1) = \frac{1}{2} + \frac{1}{2} = 1$$

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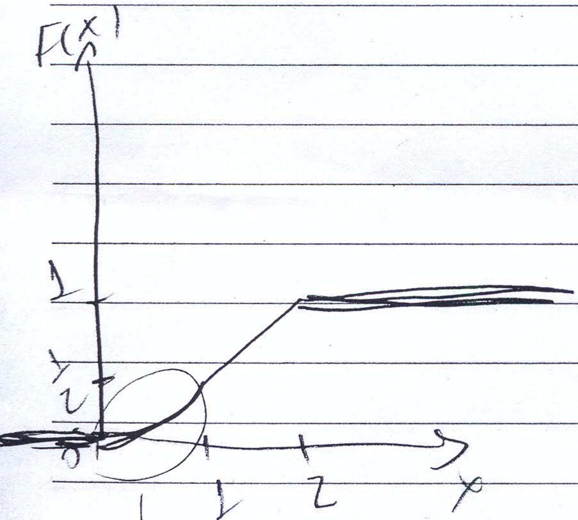
$$F(x) = \begin{cases} 0, & x < 0 \\ \dfrac{1x^2}{2}, & 0 \le x < 1 \\ \dfrac{x}{2}, & 1 \le x < 2 \\ 1, & x \geq 2 \end{cases}$$



parábola

b) $p(X<1) = F(1^-) = \frac{1}{2}$

c) $E(X) = ?$

$$E[X] = \int_{-\infty}^{+\infty} x f(x)\, dx = \int_{-\infty}^{0} x \cdot 0\, dx + \int_{0}^{1} x \cdot x\, dx + \int_{1}^{2} x \cdot \frac{1}{2}\, dx + \int_{2}^{\infty} x \cdot 0\, dx =$$

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$$= \frac{x^3}{3}\Big|_0^1 + \frac{x^2}{2\cdot 2}\Big|_1^2 = \frac{1}{3} + \frac{1}{2}\left(2 - \frac{1}{2}\right) = \frac{13}{12}$$

$E[X] = \frac{13}{12}$

$var(X) = E[X^2] - E[X]^2$

$$E(X^2) = \int_{-\infty}^{+\infty} x^2 f(x)\,dx$$

$$= \int_{-\infty}^{0} x^2 \cdot 0\,dx + \int_0^1 x^2 \cdot x\,dx + \int_1^2 x^2 \cdot \frac{1}{2}\,dx + \int_2^{\infty} x^2 \cdot 0\,dx =$$

$$0 + \int_0^1 x^3\,dx + \int_1^2 \frac{x^2}{2}\,dx + 0 =$$

$$= \frac{x^4}{4}\Big|_0^1 + \frac{1}{2}\cdot\frac{x^3}{3}\Big|_1^2 =$$

$$= \frac{1}{4} + \frac{1}{2}\left(\frac{8}{3} - \frac{1}{3}\right) = \frac{1}{4} + \frac{7}{6} = \frac{17}{12}$$

$$Var(X) = \frac{17}{12} - \frac{169}{144} = \frac{35}{144}$$

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(3) a) Modelo Poisson :

X -> VA discreta

$$P(X=K) = \frac{e^{-\lambda} \lambda^K}{K!} , \quad K \in N$$

λ = taxa média anual de tornado

$$= \frac{N^o \text{ total de tornados}}{N^o \text{anos}} = \frac{138 \text{ tornados}}{30 \text{ anos}}$$

$\boxed{\lambda = 4,6 \text{ tornados/ano}}$

b) $P(X \geq 1)$

$A = [X \geq 1]$ (evento A)

$A^c = [X < 1] \Rightarrow [X = 0]$ — $X \in N$, então o único valor que pode ter < 1 é 0

$P(A) = 1 - P(A^c)$

$$P(X=0) = \frac{e^{-\lambda} \lambda^0}{0!} = e^{-4,6} \cdot 1 = e^{-4,6}$$

$$P(X \geq 1) = 1 - e^{-4,6}$$

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c) $A \to [X > 1]$

$B \to [X < 5]$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P([X > 1] | [X < 5]) = P(A|B)$$

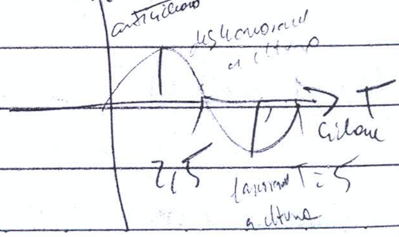
$$= \frac{e^{-1}\left(\frac{4{,}6^2}{2} + \frac{4{,}6^3}{6} + \frac{4{,}6^4}{24}\right)}{e^{-1}\left(1 + 4{,}6 + \frac{4{,}6^2}{2} + \frac{4{,}6^3}{6} + \frac{4{,}6^4}{24}\right)}$$

4) teorema do limite central

Ithaca se localiza em latitudes médias e na época do inverno, as frentes e sistemas de escala sinótica são mt importantes

1 dia escala de t

2 a 5 dias



1os

2,5 dias anti Ciclone → 2os 2,5 dias : ciclones

não dá p/ saber uma dependência das variáveis meteorológicas diárias